On the Uniform Burn-Down of Advent Wreath Candles

A combinatorial analysis with due regard for the rose candle

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Abstract

Anyone who has ever looked at an Advent wreath on Christmas Eve knows the sight: the first candle is a stub, the fourth almost new. In this paper we ask whether, assuming uniform burning, there exists a lighting schedule after which all the candles burn down equally. We show that when each Sunday is indivisible, such a schedule **does not exist**, whereas when a Sunday is split into two halves, one **does** (in fact, 480 of them). We further prove that honouring the rose Gaudete candle rules out an equal burn-down however finely time is divided. Finally, we give a simple criterion for a wreath with n candles.

Keywords: Advent wreath, divisibility, lighting schedules, Gaudete, greedy algorithm



Figure 1: The wreath under laboratory conditions.

1 The Problem

The wreath has four candles. On the first Sunday of Advent one is lit, on the second two, on the third three, and on the fourth all four. Custom suggests relighting the same "old" candle every week and adding a new one. The first candle then burns for four Sundays, the fourth for just one. The difference in height is visible to the naked eye and gives household members legitimate cause for concern.[1]

We adopt the following assumptions:

- all candles are identical and burn at a constant rate;
- candles burn only on Sundays, and every Sunday lasts equally long;
- on the k-th Sunday exactly k candles burn, but **which ones** is entirely up to us;
- draughts, cats and children with matches are neglected.[2]

The question is: can the candles be chosen so that on Christmas Eve all four are the same height?

2 Whole Sundays: Impossible

Theorem 1. *If each candle is lit for a whole Sunday or not at all, then no schedule burns the four candles down equally.*

Proof. In total the candles burn for $1 + 2 + 3 + 4 = 10$ candle-Sundays. With an equal split each would burn for $10/4 = 2.5$ Sundays. However, each candle's burning time is an integer, and 2.5 is not. □

The best achievable is the split $3, 3, 2, 2$: two candles burned one Sunday longer than the other two. Of the $4 \cdot 6 \cdot 4 \cdot 1 = 96$ possible schedules, 48 achieve this result. For example: Sunday I: A; Sunday II: A and B; Sunday III: B, C and D; Sunday IV: all of them. The customary schedule (always add a new candle) gives the split $4, 3, 2, 1$ and is the worst possible.[3]

3 Sundays in Halves: Possible

Theorem 2. *If each Sunday is divided into two equal halves and k candles burn in each of them, then there exists a schedule after which all the candles burn down perfectly evenly.*

[1] Personal observation, repeated annually for many years.

[2] The cat model is left as an open problem.

[3] Worst in terms of the difference between the longest- and shortest-burning candle. The custom does have other merits, e.g. it requires no chart on the fridge.

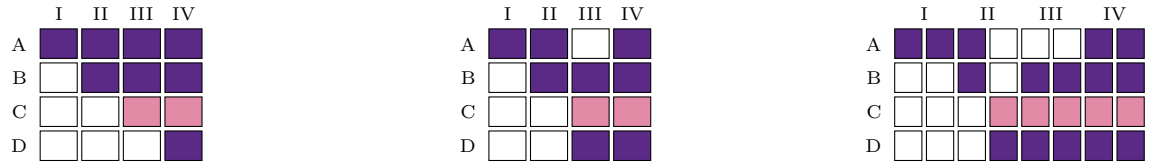


Figure 2: Lighting schedules. Filled cell: the candle burns.

Table 1: Results of the exhaustive search (wreath of 4 candles).

Sunday split	Pink waits	Admissible	Smallest spread	Optimal
whole	no	96	1 Sun.	48
whole	yes	27	1 Sun.	15
halves	no	9,216	0	480
halves	yes	1,215	1 Sun.	240
thirds	no	884,736	$\frac{1}{3}$ Sun.	131,040
thirds	yes	45,927	$\frac{2}{3}$ Sun.	1,710

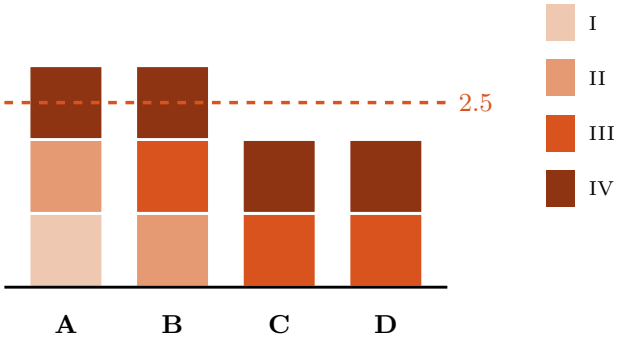


Figure 3: Proof of Theorem 1: ten candle-Sundays in four columns. The dashed line is the fair share 2.5.

Proof. There are $2 \cdot 10 = 20$ halves, so each candle should burn for 5 halves. We give such a schedule explicitly: Sunday I: A | A; II: A B | C D; III: B C D | B C D; IV: all | all. Each candle burns for exactly 5 halves. □

In practice this means that halfway through the second Sunday we snuff out A and B and light C and D. Of the 9216 admissible half-Sunday schedules, 480 are perfect. Whether the family will accept candles being snuffed out halfway through prayers lies beyond the scope of this paper.

4 The Rose Candle Lemma

The third Sunday of Advent is *Gaudete*, the Sunday of rejoicing, when the rose candle is lit for the first time. If we respect this custom, the rose candle may not be lit before the third Sunday.

Lemma 3. *If the rose candle may not burn before the third Sunday, then for* ***any*** *division of a Sunday into equal parts the candles will not burn down equally.*

Proof. The rose candle can burn at most during the third and fourth Sundays, i.e. 2 Sundays in total. An equal split requires 2.5. A finer division of time changes nothing here, since what is missing is a whole half-Sunday, not some fraction of a part. □

So what can be done? Without division, the best is again the split $3, 3, 2, 2$, with the rose candle burning for 2 Sundays (15 such schedules out of 27). With halves, the best outcome is 3; 2.5; 2; 2.5, so the total deviation is half that without division. With a split into thirds, something beautiful happens: the three purple candles burn equally, $2\frac{2}{3}$ Sundays each, while the rose one burns only 2.

Pastoral corollary. The rose candle always ends up the tallest. Joy outlasts penance.[4]

5 Generalisation: a Wreath with n Candles

Consider a wreath with n candles and n Sundays, in which each Sunday is divided into m equal parts, and in each part of the k-th Sunday k candles burn.

Theorem 4. *An equal split exists if and only if* $m(n+1)$ *is even.*

Proof. There are $m \cdot \frac{n(n+1)}{2}$ "candle-parts" in total, so each candle gets $\frac{m(n+1)}{2}$. This must be an integer, hence the necessary condition. Sufficiency follows from a greedy strategy: in each part we light the k candles that have burned the least so far. If before a given part the burning times differ by at most 1, then they still do afterwards (we raise the lowest ones). At the end the total is divisible by n and the values differ by at most 1, so they are all equal. □

An amusing fact follows: a wreath with **three** or **five** candles could be burned down evenly without splitting any Sundays at all. Four candles are, mathematically speaking, the least convenient choice.[5]

[4] The theological thesis was not subject to peer review.

[5] We are not proposing any change to tradition. Merely ob-

Table 2: Does an equal split exist for n candles and m parts per Sunday (✓ — yes).

$n\backslash m$	1	2	3	4	5	6
2	·	✓	·	✓	·	✓
3	✓	✓	✓	✓	✓	✓
4	·	✓	·	✓	·	✓
5	✓	✓	✓	✓	✓	✓
6	·	✓	·	✓	·	✓
7	✓	✓	✓	✓	✓	✓
8	·	✓	·	✓	·	✓

6 Conclusions

- Without splitting Sundays, burning four candles down evenly is impossible. The best result is $3, 3, 2, 2$.
- With Sundays split into halves it is possible, in 480 ways.
- If the rose candle waits for Gaudete, an even burn-down cannot be achieved by any division of time.
- In the general case it suffices to check whether $m(n+1)$ is even.



Figure 4: Experimental verification (Christmas Eve, half-Sunday variant).

A final piece of practical advice: if you care about even candles, light the tallest ones each Sunday. This is precisely the greedy strategy of Theorem 4, and it works even when nobody keeps a chart.

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Replication code: `https://czteryswiece.ponadmurami.pl/en/wieniec/`

References

1. J. H. Wichern, the first Advent wreath at the Rauhes Haus, Hamburg 1839 (historical account; according to tradition it had a candle for every day of Advent).
2. Universal Norms on the Liturgical Year and the Calendar, 1969, nos. 39–42 (Advent has four Sundays).
3. P. G. L. Dirichlet, the pigeonhole principle, 19th century (cited here more out of sentiment than necessity).

serving.